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TRANSFER EFFECTS IN ISOMORPHIC PROBLEM SITUATIONS*

George F. LUGER and Michael A. BAUER**
Dept. of Artificial Intelligence, University of Edinburgh, Great Britain

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The results of a recent experiment to assess the relationship between human problem solving behaviour and the structural properties of certain problems are presented. The results suggest that human performance in a problem-solving task is significantly influenced by immediately prior experience on a different but structurally identical problem. Specifically, transfer effects are demonstrated across two problems of isomorphic structure.

1. Introduction

A large class of problems may be formally described by a state-space representation (Nilsson 1971; Goldin and Luger 1975). In this representation each different problem configuration is represented by a single state; a start state represents the initial configuration and goal state(s) represent the possible goal configurations of the problem. Moves from state to state in the representation correspond to the legal moves that transform one problem configuration into another. A solution to the problem is expressed as a path through the state-space from the start state to a goal state. Using this representation notions such as problem decomposition, problem extension, and the relationships between several problems, for example, isomorphisms and homomorphisms, may be clearly defined (Banerji 1969; Banerji and Ernst 1972; Luger 1976).

This paper presents the results of an experiment to assess the relationship between human problem solving behaviour and the struc-

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** Current address: Dept. of Computer Science, University of Western Ontario, London, Ontario, Canada.

tural properties of such problems. In particular, these results suggest that human performance in a problem solving task may be significantly influenced by immediately prior experience on a different but isomorphic, i.e., structurally identical, problem.

Although much recent research has considered the more general questions of relating problem structure to problem-solving behaviour (Newell and Simon 1972; Egan and Greeno 1974) three recent studies (Reed et al. 1974; Thomas 1974; Hayes and Simon 1975) have explored problem solving behaviour in specific tasks.

Reed et al. (1974) designed a study on the well-known Missionaries and Cannibals (MC) problem:

> Three missionaries and three cannibals wish to cross a river. They find a boat but it is so small that it can only contain two people. If the missionaries on either bank of the river are outnumbered at any time by cannibals they will be eaten. Devise a schedule of crossings that will permit all missionaries and all cannibals to cross the river safely. It is assumed that all passengers on the boat unload before the next trip begins, and that at least one person is in the boat for each crossing.

Reed et al. then describe a homomorph of the MC problem called 'Jealous Husbands' (JH) in which missionaries are replaced by husbands and cannibals by wives with the additional constraint that husbands and wives must be paired. In one experiment, subjects performed MC then JH, (MC 1 then JH 2); a second group performing JH then MC (JH 1 then MC 2). Subjects were required to solve each problem, in order, once and to solve both problems within a 30 min time limit. No significant reduction in time, total number of moves or number of illegal moves was observed between JH 1 and JH 2 or between MC 1 and MC 2.

In a second experiment one group solved MC twice (MC 1 then MC 2), and a second group solved JH twice (JH 1 then JH 2). Here significant improvement in time was found only for the JH problem (< 0.01), although a reduction in the number of illegal moves was observed for both problems (< 0.05). The third experiment was like the first except that subjects were told the relationship between the two problems. Only performance for JH 1-MC 2 improved (< 0.01 for both time and number of illegal moves).

Thomas (1974) designed an experiment using an isomorph of the MC problem called the 'Hobbits and Orcs' problem (missionaries are replaced by hobbits and cannibals by orcs). A control group solved the problem once and an experimental group solved the problem twice: first from a state in the middle of the problem and then from the initial state. The combined number of legal and illegal moves required to solve the first part of the problem (through the first five states of the problem to where one Missionary and one Cannibal are on the start side of the river) decreased (< 0.05) for the experimental group.

Hayes and Simon (1975) designed their study to analyse the differences in transfer effects caused by variations among the texts of isomorphic 'monster' problems. One variant (with monsters using either transfer or change operators and being either agent or patient of the operator) of a set of isomorphic problems was given to each group of subjects. This was followed by a second (non-isomorphic) problem from another set of isomorphic problems having the same variations. The transfer-change and agent-patient variables were held constant across each group. Problems involving transfer operators were solved much more quickly than those involving change operator. All variations influenced the subjects' notation or the representation (and so the 'problem space', Newell and Simon 1972) employed in solving the problems.

The Reed et al. and the Thomas research investigated the relationship of a problem's formal structure and the behaviour of subjects solving the problem. However, we felt that some points needed further exploration. First, the difficulty of the MC problem does not seem to originate from the complexity of the state-space (which contains fifteen states and four possible solution paths) but rather from the difficulty of transforming one state into another, that is, in discovering legal moves. This raised the question of whether there would be similar effects if problems were considered where the transformations from state to state (that is, the legal moves) were simpler and the difficulty of the problem rested on selecting one transformation from among several at each step of the solution path.

Further, Thomas found improved performance in part/whole problem solving and Reed et al. found some improvement in their second experiment (that is, more practice on the same problem brought higher levels of performance). We felt an interesting complement to these studies would be to consider subjects' behaviour across problems of identical structure. Reed et al. found no improvement on different though homomorphically related problems (unless subjects were made explicitly aware of the relationships, and then only in the JH 1 then

MC 2 problem order). Again, we felt consideration should be given to the performance of subjects across isomorphic problems. Finally, in line with the Hayes and Simon findings we have focused on the 'agent-transfer' class of problems, but, in contrast with their work, we used more complex problems (more than twice as many states) and also tested *across* isomorphic problems.

2. The experiment

2.1. The problems

In the Tower of Hanoi problem four concentric rings (labelled 1, 2, 3, 4 respectively) are placed in order of size, the largest on the bottom, on the first of three pegs (labelled A, B, C); the apparatus is pictured in fig. 1. The object of the problem is to transfer all the rings from peg A to peg C in the fewest possible number of moves. There are two constraints: only one ring–the top ring on any peg–may be moved at a time, and no larger ring may be placed over a smaller one on any peg.

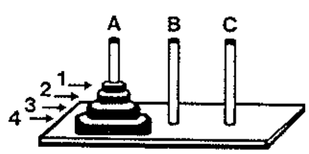

Fig. 1. The Tower of Hanoi problem in its initial state. 1, 2, 3, 4 and A, B, C illustrate the TOH/TC isomorphism.

The Tea Ceremony (see fig. 2) is an isomorph of the Tower of Hanoi, first studied by Hayes and Simon (1974). Three people, a host, an elder and a youth, participate in the ceremony. There are four tasks they perform, listed in ascending order of importance: feeding the fire, serving cakes, serving tea, and reading poetry. The host performs all the tasks at the beginning of the ceremony, and the tasks are transferred back and forth among the participants until all the tasks are performed by the youth, at which time the ceremony is completed. There are two constraints on the transfer of tasks: only one task–the least important a person is performing–may be moved, and no person may receive a new task unless it is less important than any task they perform at the time. The object of the Tea Ceremony game is to transfer the four tasks from the host to the youth in the fewest number of moves.

In the Tea Ceremony each task is represented by blocks with a 1.5 inch square base. The height of each block (0.5, 1, 1.5, and 2 inches) represents the relative importance of each task. The transfer of each task is constrained by a 0.25 inch high track. This allows–as do the rings stacked on pegs in the Tower of Hanoi–

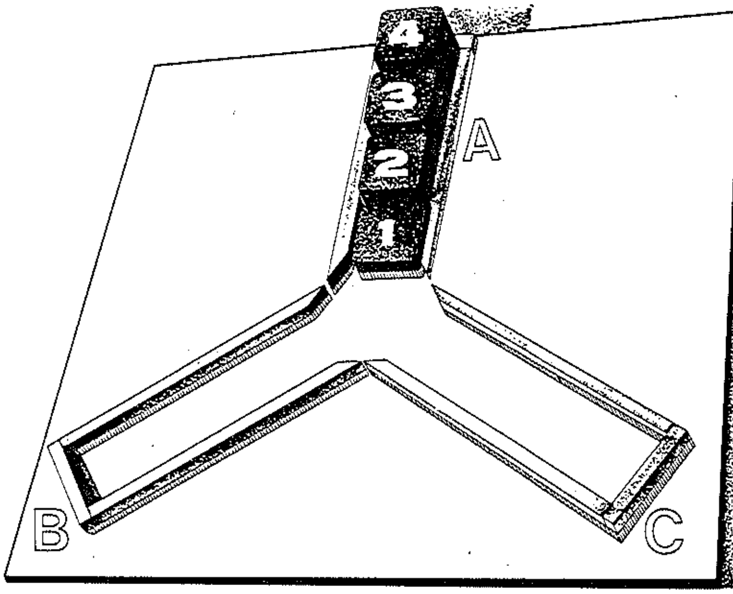

Fig. 2. The Tea Ceremony problem in its initial state. 1, 2, 3, 4 and A, B, C illustrate the TOH/TC isomorphism. (For testing purposes, FIRE, CAKES, TEA, POETRY, HOST, ELDER, and YOUTH replace 1, 2, 3, 4, A, B, and C respectively).

access only to the least important task a person is performing while at the same time allowing the illegal move of transferring to people tasks more important than those they already perform.

In the isomorphic relationship between the Tower of Hanoi and the Tea Ceremony the people—host, elder, and youth—correspond respectively with pegs A, B, and C. The four tasks—feeding the fire, serving cakes, serving tea, and reading poetry—correspond respectively with rings 1, 2, 3, and 4. It can be shown that the initial state, goal state, and set of legal moves of the two games correspond.

Fig. 3 is the complete state-space representation of the Tower of Hanoi/Tea Ceremony problem. Each circle stands for a possible position or state of the problem. The four letters labelling a state refer to the respective pegs (people) on which the four rings (tasks) are located. For example, state CCBC means that ring 1 (fire), ring 2 (cakes), and ring 4 (poetry) are in their proper order on peg C (performed by the youth). Ring 3 (tea) is on peg B (performed by the elder). A legal move by the problem solver always effects a transition between states represented by neighbouring circles in fig. 3. The solution path containing the minimum number of moves consists of the 15 steps from AAAA to CCCC down the right hand side of the state-space diagram.

In contrast to the MC/JH, the difficulty of the TOH/TC problem seems to lie in discovering a sequence of legal moves. Transferring the rings from peg A to peg C (or services from Host to Youth) involves the solution of several intermediate

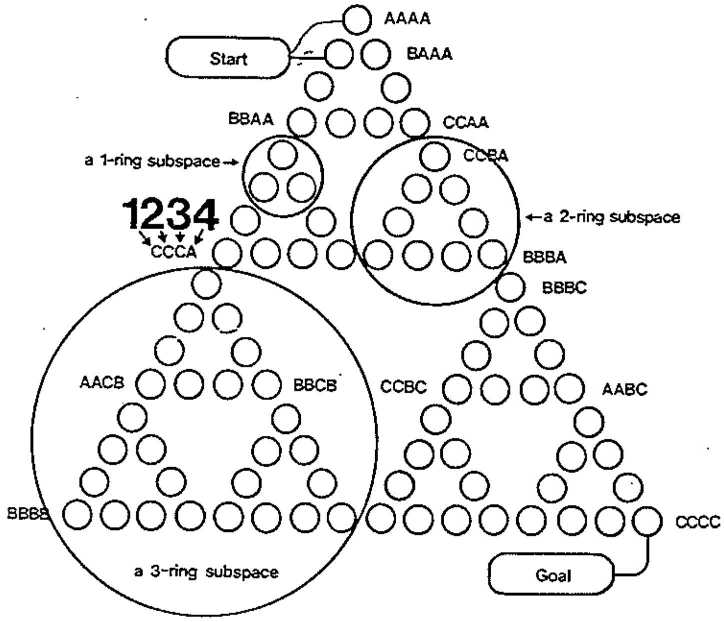

Fig. 3. The TOH/TC state space. Note examples of 1-, 2-, and 3-ring subspaces.

subproblems. This problem and subproblem structure is readily observable in the state-space (Simon 1975; Luger 1976) *S*s have little trouble discerning legal moves or possible sequences of legal moves. In fact, in a prestudy (Luger 1973) the subjects took an average of about 5 sec to make each legal move.

2.2. Method

Forty-eight subjects, all second-year Psychology students at the University of Edinburgh, volunteered for the study. These were randomly placed in two treatment groups: 24 *S*s solving the Tower of Hanoi problem first and then the Tea Ceremony (TOH 1 then TC 2) and 24 *S*s solving the Tea Ceremony first and then the Tower of Hanoi (TC 1 then TOH 2). In this way the TOH 1 data was used as control for the TOH 2 results and TC 1 data for the TC 2 results (see Reed et al. 1974).

The data of six *S*s was not used either because of prior experience with the problems (2) or an inability to meet criterion on both problems within a 45 min time period (4). The *S*s were asked to work on the problems one at a time. Criterion for each problem consisted in the *S*s performing the moves of the minimal solution within a 90 sec. time period. Although the *S*s were not explicitly told of the time criterion, they became aware of some constraint by the researcher's statement after a minimal solution taking longer than 90 sec: "Your solution was in the fewest possible number of moves. Now see if you can repeat this solution a little more quickly". There were two reasons for selecting the 90 sec minimal solution as criterion: (1) the researchers wanted to avoid 'overlearning', i.e., the reinforcement of the solution steps by several repetitions (it was thought this reinforcement would be unnecessary for transfer), and (2) previous research (Luger 1973, 1976) had demonstrated that *S*s satisfying the 90 sec criterion could repeat their solution in about the same amount of time.

No reference was made, either before or during the problem solving, to the relationships between the two problems. In fact, the *S*s were told they would be solving two different problems, and while one was being solved the other problem was always out of sight. Only after solving both problems were the *S*s asked if and when they noticed any relationship between the problems.

Since the *S* was required to solve each problem in the minimum number of moves, the problem could be started again any time the *S* 'got lost' or 'saw a better way'. This was done by the *S* simply placing the pieces back in the initial position and starting again. If the *S* took more than the minimum number of moves to reach the goal state on completion of the problem, the pieces were returned to the start state, and the *S* was asked to try to solve the problem again using fewer moves.

The *S*s were tested one at a time by one of the authors. A tape recorder was used to record any comments the *S* might make and to record the *S*s' moves. This was used later to determine the move sequences and the time elapsed between moves.

2.2.1. Hypotheses tested

The primary aim of the study was to test the improvement in performance of *S*s solving two problems of isomorphic structure. The two measures of performance were the total time required and the total number of problem states entered by *S*s solving each problem (i.e., in producing one minimal state solution path within a 90 sec time interval). For completeness in comparison with the Reed et al. and with the Thomas studies, the number of illegal moves was also compared across problems. Thus, the comparison of TOH 1 with TOH 2 and TC 1 with TC 2 on total time required, states entered, and illegal moves attempted made up the hypotheses of the study.

2.3. Results

The data for each *S* is summarized in table 1 by recording the medians and interquartile ranges of time required, total number of states entered, and the number of illegal moves attempted in solving each problem. The Mann-Whitney U-test was used to compare the distributions of TOH 1 with TOH 2 and TC 1 with TC 2 on each of the measures (times, states, and illegal moves).

The U-test revealed significant decrease in total time and total states entered between TOH 1 and TOH 2 ($p < 0.001$ for both time and states using a one-tailed test) and between TC 1 and TC 2 ($p < 0.02$ for time and $p < 0.05$ for states, using a one-tailed test). The difference in number of illegal moves between the problems turned out to be so small that it was hardly relevant. Thus, the null hypothesis that there was no difference between distributions of TOH 1 and TOH 2 on total time required and problem states entered in solving each problem was rejected. Similarly,

Table 1
The medians (M) and inter-quartile ranges (IQR) for each group of *S*s and each problem. The (one-tailed) Mann-Whitney U-test was used to determine the significance levels.

	TOH1 (n=23)		TOH2 (n=19)		TOH1=TOH2
Time (sec)	M = 307	IQR = 233	M = 79	IQR = 134	< 0.001
Number of states	M = 69	IQR = 26	M = 32	IQR = 30	< 0.001
Illegal moves	M = 0	IQR = 0	M = 0	IQR = 0	–
	TC1 (n=19)		TC2 (n=23)		TC1=TC2
Time (sec)	M = 386	IQR = 408	M = 242	IQR = 245	< 0.02
Number of states	M = 88	IQR = 114	M = 61	IQR = 68	< 0.~~001~~ ,05
Illegal moves	M = 1	IQR = 2	M = 0	IQR = 1	–

the null hypothesis on the distributions of TC 1 and TC 2 was rejected.

The analysis of variance (table 2) offers an even fuller appreciation of the data. Here the transfer effect is suggested by a significant order by problem interaction. This was significant ($p < 0.01$) for both time required and states entered in solving the problems. The problem order did not have a significant effect on either measure. That is, the *total* number of moves and *total* time required for solving *both* problems was not influenced by which problem was solved first. The problem type, however, was significant ($p < 0.01$). This indicates (see also table 1) that the Tea Ceremony problem required a larger amount of time and greater number of states regardless of whether it was solved first or second.

Table 2
The analysis of variance for time required and states entered by *S*s solving the Tower of Hanoi and Tea Ceremony's problems.

	Factor	*df*	*MS*	*F*
Time required				
	Problem type	1	325535	7.34**
	Problem order	1	1667	0.04
	Subjects (nested within order)	36	53977	1.22
	Intn. type/order	1	779322	17.57**
	Error	36	44353	
States entered				
	Problem type	1	14645	21.8**
	Problem order	1	128	0.19
	Subjects (nested within order)	36	78179	3.23*
	Intn. type/order	1	22754	33.9**
	Error	36	671	

* $p < 0.05$.
** $p < 0.01$.

3. Discussion

The result of this study was that strong evidence was found for transfer between the two different tasks of identical structure. Furthermore, these effects occurred without subjects being made explicitly aware of the structural relationship existing between the problems. In fact, a majority of the subjects could only describe the full relationship after it had been pointed out by one of the authors at the conclusion of the problem-solving session.

There may be several explanations of the fact that our data showed transfer effects while that of the Reed et al. study did not. First, the MC/JH state-space (as pointed out in the introduction and by both Reed et al. and Thomas) did not reflect the complexity of the problem environment as seen by the subject, that is, the difficulty of *selecting* legal moves is not reflected in the state-space. Thus the calculations of numbers of legal moves and time required in crossing the MC/JH state-space does not seem to be an altogether reliable model of the information processed by the problem-solving subjects. Further, almost one in every six moves attempted by subjects in the MC/JH study was illegal and, as such, not describable as a move within the MC/JH state-space. The TOH/TC subjects, on the other hand, had only about one illegal move for every one hundred moves attempted and used about five seconds per move as opposed to 15 seconds per move for the MC/JH subjects. In these respects the TOH/TC state-space seems to offer a much more reliable record of subjects' problem-solving behaviour.

Secondly, the homomorphic relationship between JH and MC was not reflected in the states of the state-space: rather it was a mapping on the operators for generating moves between states. There is, in fact, a one-to-one mapping between the legal moves of the two problems. This form of homomorphic relationship may be too weak to expect a significant reduction in the number of problem states entered by subjects in solving each problem. It seems necessary that if problem-solving behaviour is to be recorded as intervals of time and moves through a state-space, care must be taken that, as far as possible, all states of the game environment considered distinct by the problem solver are represented as distinct in the space.

A further comment relating the results of these studies is that subjects' abilities to generate legal moves as well as to utilize a problem's structure and its morphological relationships with the structures of other problems can only partially explain transfer effects: the nature of the problem structure itself is important. For example, the TOH/TC problem has an interesting substructure of nested isomorphic subproblems. A successful solver of either the TOH or TC problem, while producing within an 81 state state-space a minimal solution sequence of 15 moves, will effectively decompose the problem into its subproblems and may experience the overall problem symmetry (Luger 1976). The total of this experience is brought to the next problem faced. The MC/JH problem, on the other hand, with only 15 problem states and four different 11-move solution paths, seems to lack an interesting substructure. This fact could help explain the lack of transfer in the Reed et al. study.

There seem to be several areas open to future study. One question might be to examine transfer in the homomorphic problem situations using state spaces that reflect the homomorphic relationship. The 3- and 4-ring Tower of Hanoi and 3- and 4-service Tea Ceremony problems might offer a suitable domain for this study. Further research should also separate the effects due to game playing familiarity from the effects due to the morphological relationships between the problems, for example, by comparing subjects solving the Missionary and Cannibal and then the Tea Ceremony problems with those solving the Tower of Hanoi and then the Tea Ceremony problems. Finally, research must also focus more closely on features of a problem's structure (such as its possible sub-problem and symmetry decompositions) in an attempt to determine which and how much each effects transfer. Simon (1975) has begun this study, and the authors are currently considering this problem in the light of the TOH/TC data.

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